Maxwell's equations and accelerated frames

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To analyze electromagnetism in spinning media, we use, differently than previous works, a relativistic description of rotations. We give the form of Maxwell's equations in the laboratory and corotating frames in terms of cylindrical coordinates. Possible applications are discussed. [S1063-651X(98)08106-9]

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I. INTRODUCTION

It has been known for a long time that Maxwell's equations are covariant under relativistic transformations, but the form of the constitutive relations necessary to get a determinate system of equations still seems to be controversial, particularly in accelerated media [1–6]. We discuss here this question for uniformly rotating media, which have been the subject of many works [4,7–9].

For accelerated media, one has to deal with three different frames: the laboratory frame K_L, the corotating frame K_C, and the instantaneous inertial frame K' in which the material medium is at rest. We use the cylindrical coordinates $X^\mu(R,\Phi,Z,X^0=cT)$ in K_L, $x^\mu(r,\phi,z,x^0)$ in K_C, and X'^μ in K'. The greek (latin) indices take the values 0,1,2,3 (1,2,3) and we use the summation convention.

Previous works on electromagnetism in rotating media [4,7–9] describe the rotation by the Galilean transformation

$$r=R,\quad \phi=\Phi-\Omega c^{-1}X^0,\quad z=Z,\quad x^0=X^0, \tag{1}$$

As a consequence the relativistic covariance is broken; this failure is interpreted wrongly as a noninertial local effect of rotation. It was proved recently [10] that the relativistic Trocheris-Takeno transformation [11,12]

$$r=R,\quad \phi=(\cosh\beta)\Phi-R^{-1}(\sinh\beta)X^0,\quad z=Z,$$
$$x^0=-R(\sinh\beta)\Phi+(\cosh\beta)X^0, \tag{2}$$

with $\beta=\Omega R/c$, restores the full Lorentz covariance of electromagnetism. As noticed in [10], the transformation (2) "respects the relativity of simultaneity at a distance, the relativistic law of composition of velocities, and the additive law of angular velocities." The speed-distance law is nonlinear as the three-velocity, which is now

$$v_R=v_Z=0,\quad v_\Phi=c\tanh\beta. \tag{3}$$

We work in this paper with the transformation (2) and incidentally the Sagnac phase shift ΔT becomes, with Eq. (2),

$$\Delta T=4\pi c^{-1}r\tanh\beta(1-\tanh^2\beta)^{-1}=2\pi c^{-1}r\sinh 2\beta, \tag{4}$$

which reduces for $\beta\ll 1$ to the expression obtained with Eq. (1) [9].

We consider in this work a linear, scalar medium, in uniform rotation with angular velocity ω and assume that in the inertial frame K' where this material medium is at rest, the constitutive relations have their more general forms [1,13,14]

$$\mathbf{D}'=\varepsilon\mathbf{E}'+\alpha\mathbf{H}',\quad \mathbf{B}'=\mu\mathbf{H}'+\lambda\mathbf{E}'. \tag{5}$$

$\mathbf{E}',\mathbf{D}',\mathbf{H}',\mathbf{B}'$ are the usual components of the electromagnetic field, ε,μ the permittivity and permeability, and α,λ the chiral parameters. It is assumed that rotation does not change the physical properties of this medium. Using Eqs. (2) and (5), we now have to get the constitutive relations in the K_L and K_C frames.

II. FRENET-SERRET AND LABORATORY FRAMES

As just said, the constitutive relations (5) remain valid in the instantaneous inertial frame K' in which the material medium is at rest and it is known [4,10] that K' is the Frenet-Serret tetrad made of four unit vectors e'_μ defined in the following way [15]. The four-velocity u^μ is taken as a timelike unit vector e'_0 and the spacelike vectors e'_k are obtained from the relations

$$\partial/\partial s e'_0=ae'_1,\quad \partial/\partial s e'_1=be'_2+ae'_0,$$
$$\partial/\partial s e'_2=ce'_3-be'_1,\quad \partial/\partial s e'_3=-ce'_2. \tag{6}$$

The coefficients a,b,c are non-negative, $\partial/\partial s$ is the absolute derivative, and for a four-vector A^μ

$$\partial/\partial s A^\mu=u^\beta\partial_\beta A^\mu+\Gamma^\mu_{\alpha\beta}A^\alpha u^\beta. \tag{7}$$

$\partial_\mu=\partial/\partial x^\mu$ and the parameters $\Gamma^\mu_{\alpha\beta}$ are the Christoffel symbols deduced here from the metric of Minkowski space-time in cylindrical coordinates

$$ds^2=(dX^0)^2-(dR)^2-R^2(d\Phi)^2-(dZ)^2. \tag{8}$$

According to Eq. (3), the components of the four-velocity u^μ are

$$u^0=\cosh\beta,\quad u^2=R^{-1}\sinh\beta,\quad u^1=u^3=0. \tag{9}$$

Let $e^\beta_\mu=\delta^\beta_\mu$ denote the coordinate covariant basis vectors in the laboratory frame and δ^β_μ the Kronecker symbol. Then a simple calculation gives [10] $e'_1=e_1$, $e'_3=e_3$, and

$$e'_0=(\cosh\beta)e_0+R^{-1}(\sinh\beta)e_2, \tag{10}$$

$$e_2' = (\sinh\beta)e_0 + R^{-1}(\cosh\beta)e_2.$$

It is easy to check that the transformations (10) allow the metric (8) to remain invariant, restoring, as previously mentioned, the full Lorentz covariance so that Maxwell's equations have the same form in both frames K_L and K'. However, one has the same relations between the electromagnetic components **E**,**D**,**H**,**B** in the K_L frame and the corresponding primed components in the instantaneous inertial corotating frame K' as one would have between two inertial frames, except of course that the velocity **v** is no longer constant. So we have [9,16], with $\gamma = (1 - v^2 c^{-2})^{-1/2}$,

$$\begin{aligned}
\mathbf{E}' &= E_\parallel + \gamma(E_\perp + c^{-1}\mathbf{v}\wedge\mathbf{B}),\\
\mathbf{B}' &= B_\parallel + \gamma(B_\perp - c^{-1}\mathbf{v}\wedge\mathbf{E}),\\
\mathbf{D}' &= D_\parallel + \gamma(D_\perp + c^{-1}\mathbf{v}\wedge\mathbf{H}),\\
\mathbf{H}' &= H_\parallel + \gamma(H_\perp - c^{-1}\mathbf{v}\wedge\mathbf{D}).
\end{aligned} \tag{11}$$

The subscripts $\parallel$ and $\perp$ refer to components parallel and perpendicular to **v**, respectively. From Eqs. (5) and (11) we get

$$\begin{aligned}
&\mathbf{D} + c^{-1}\mathbf{v}\wedge\mathbf{H} = \varepsilon(\mathbf{E} + c^{-1}\mathbf{v}\wedge\mathbf{B}) + \alpha(\mathbf{H} - c^{-1}\mathbf{v}\wedge\mathbf{D}),\\
&\mathbf{B} - c^{-1}\mathbf{v}\wedge\mathbf{E} = \mu(\mathbf{H} - c^{-1}\mathbf{v}\wedge\mathbf{D}) + \lambda(\mathbf{E} + c^{-1}\mathbf{v}\wedge\mathbf{B}).
\end{aligned} \tag{12}$$

Substituting the velocity (3) into Eq. (12) and rearranging the terms, a simple calculation gives, with $s = \tanh\beta$,

$$D_\Phi = \varepsilon E_\Phi + \alpha H_\Phi, \quad B_\Phi = \mu H_\Phi + \lambda E_\Phi; \tag{13}$$

$$\begin{aligned}
D_R + \alpha s D_Z - \varepsilon s B_Z &= F_R = \varepsilon E_R + \alpha H_R - s H_Z,\\
D_Z - \alpha s D_R + \varepsilon s B_R &= F_Z = \varepsilon E_Z + \alpha H_Z + s H_R,\\
B_R - \lambda s B_Z + \mu s D_Z &= G_R = \mu H_R + \lambda E_R + s E_Z,\\
B_Z + \lambda s B_R - \mu s D_R &= G_Z = \mu H_Z + \lambda E_Z - s E_R.
\end{aligned} \tag{14}$$

We introduce the matrices

$$M = \begin{vmatrix} M_1 & M_2 \\ M_3 & M_4 \end{vmatrix}, \quad N = \begin{vmatrix} M_4 & -M_2 \\ -M_3 & M_1 \end{vmatrix},$$

$$I = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \quad J = \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} \tag{15}$$

such that

$$M_1 = I + \alpha s J, \quad M_2 = -\varepsilon s J, \quad M_3 = \mu s J, \quad M_4 = I - \lambda s J. \tag{15$'$}$$

We write Eq. (14) in the matrix form

$$MA = P, \quad A = |D_R, D_Z, B_R, B_Z|^t, \quad P = |F_R, F_Z, G_R, G_Z|^t, \tag{16}$$

in which the superscript t denotes transposition. Then, multiplying Eq. (16) by the matrix N and noting that the submatrices M_j $(j = 1,2,3,4)$ commute, we get

$$(M_1M_4 - M_2M_3)\otimes IA = NP, \tag{17}$$

with $(n^2 = \varepsilon\mu)$

$$M_1M_4 - M_2M_3 = \begin{vmatrix} 1 + (\alpha\lambda - n^2)s & (\alpha - \lambda)s \\ (1-a)s & 1 + (a\lambda - n^2)s^2 \end{vmatrix}. \tag{17$'$}$$

Note that this matrix is diagonal for $\lambda = \alpha$. Since the inversion of Eq. (17′) is trivial, we get finally from Eq. (17) the constitutive relations in the laboratory frame

$$A = (M_1M_4 - M_2M_3)^{-1}\otimes INP. \tag{18}$$

For an isotropic medium $(\alpha = \lambda = 0)$, we get easily from Eq. (14)

$$\begin{aligned}
(1 - n^2s^2)D_R &= \varepsilon(1 - s^2)E_R - s(1 - n^2)H_Z,\\
(1 - n^2s^2)D_Z &= \varepsilon(1 - s^2)E_Z + s(1 - n^2)H_R,\\
(1 - n^2s^2)B_R &= \mu(1 - s^2)H_R + s(1 - n^2)E_Z,\\
(1 - n^2s^2)B_Z &= \mu(1 - s^2)H_Z - s(1 - n^2)E_R.
\end{aligned} \tag{19}$$

For $\beta \ll 1$ so that $s = \beta + O(\beta^2)$, these expressions become the constitutive relations obtained with the Galilean transformation (2) [9].

III. ELECTROMAGNETIC FIELD IN THE COROTATING FRAME

The situation is a bit more intricate in the corotating frame since one has in fact to use the general covariant formalism of electromagnetism. From the Trocheris-Takeno transformation (2) we get $dR = dr$, $dZ = dz$, and

$$\begin{aligned}
dX^0 &= (\cosh\beta)dx^0 + (\sinh\beta)r\,d\phi + a_0\,dr,\\
R\,d\Phi &= (\cosh\beta)r\,d\phi + (\sinh\beta)dx^0 + a_2\,dr;
\end{aligned} \tag{20}$$

$$\begin{aligned}
a_0 &= (\sinh\beta)(\phi + \beta x^0/r) + (\cosh\beta)(\beta\phi),\\
a_2 &= (\sinh\beta)(\beta\phi - x^0/r) + (\cosh\beta)(\beta x^0/r).
\end{aligned} \tag{20$'$}$$

Substituting Eq. (20) into Eq. (8), a simple calculation gives the metric ds^2 in the corotating frame K_C,

$$\begin{aligned}
ds^2 = {}&(dx^0)^2 - \lambda_1^2(dr)^2 - r^2(d\phi)^2 - (dz)^2 + 2\lambda_0\,dr\,dx^0\\
&+ 2\lambda_2\,r\,dr\,d\phi,
\end{aligned} \tag{21}$$

$$\lambda_1^2 = 1 + a_2^2 - a_0^2,$$

$$\begin{aligned}
\lambda_0 = a_0(\cosh\beta) - a_2(\sinh\beta), \quad \lambda_2 = {}&a_0(\sinh\beta)\\
&- a_2(\cosh\beta).
\end{aligned} \tag{21$'$}$$

So the metric tensor $g_{\mu\nu}$ is

$$g_{\mu\nu} = \begin{vmatrix} 1 & \lambda_0 & 0 & 0 \\ \lambda_0 & -\lambda_1^2 & r\lambda_2 & 0 \\ 0 & r\lambda_2 & -r^2 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} \tag{22}$$

and we get easily $|g| = -\det g_{\mu\nu} = r^2$.

We may now use the general covariant formalism of electromagnetism [9]. Let $F^{\mu\nu}(\mathbf{e},\mathbf{b})$ and $G^{\mu\nu}(\mathbf{d},\mathbf{h})$ be the two antisymmetric tensors characterizing the electromagnetic field and satisfying the covariant Maxwell equations

$$|g|^{-1/2}\partial_\beta(|g|^{1/2}G^{\mu\beta})=0,$$
$$\partial_\gamma F_{\alpha\beta}+\partial_\alpha F_{\beta\gamma}+\partial_\beta F_{\gamma\alpha}=0. \tag{23}$$

If we further impose that the components $G^{0i},G^{kl},F^{0i},F^{kl}$ are three-vectors (a discussion of this condition is given in [9]) we have

$$G^{0j}=d^j,\quad G^{jk}=|g|^{-1/2}\varepsilon^{jkl}h_l,$$
$$F_{0j}=-e_j,\quad F_{jk}=(|g|/g_{00})^{1/2}\varepsilon_{jkl}b^l. \tag{24}$$

$\varepsilon_{jkl}=\varepsilon^{jkl}$ is the permutation tensor. Since $g_{00}=1$ and $|g|=r^2$, for the metric tensor (22) these relations become

$$D^R=d^r,\quad D^\Phi=d^\phi,\quad D^Z=d^z;$$
$$H_R=r^{-1}h_r,\quad H_\Phi=r^{-1}h_\phi,\quad H_Z=r^{-1}h_Z;$$
$$E_R=e_r,\quad E_\Phi=e_\phi,\quad E_Z=e_z;$$
$$B^R=rb^r,\quad B^\Phi=rb^\phi,\quad B^Z=rb^z. \tag{25}$$

Substituting Eqs. (25) into Eqs. (13) and (14) yields the constitutive relations in the corotating frame. For instance, using the contravariant components of the fields on the left-hand side of Eqs. (14) the first equation of this system becomes

$$D^R+\alpha sD^Z+\varepsilon sB^Z=-(\varepsilon E_R+\alpha H_R-sH_Z). \tag{26}$$

Substituting Eqs. (25) into Eq. (26) gives

$$d^r+\alpha sd^z+\varepsilon srb^z=-(\varepsilon e_r+\alpha r^{-1}h_R-sr^{-1}h_z). \tag{27}$$

We have similar relations for the other three equations (14) and to eliminate b or d from the corresponding equations we would proceed as in Sec. II.

Inserting Eqs. (25) into Eqs. (23), the first set of Maxwell's equations in the corotating frame is

$$r^{-1}(\partial_\phi e_z-\partial_z e_\phi)=-\partial_{x^0}b^r,$$
$$\partial_z e_r-\partial_r e_z=-\partial_{x^0}(rb^\phi), \tag{28}$$
$$r^{-1}(\partial_r e_\phi-\partial_\phi e_r)=-\partial_{x^0}b^z,$$

$$r^{-1}[\partial_r(rb^r)+\partial_\phi b^\phi]+\partial_z b^z=0. \tag{28$'$}$$

Changing e and b, respectively, into h and $-d$ in Eqs. (28) and (28$'$) yields the second set of Maxwell's equations.

It should be noted that the components of the electromagnetic field defined by relations (25) do not have their usual form in cylindrical coordinates. To get these usual components one has [9] to use the metric tensor $\gamma_{ik}=g_{0i}g_{0k}g_{00}^{-1}-g_{ik}$ of the three-dimensional space of the corotating frame. Let a_i (a^i) denote any component (25). Then the ordinary component $a_{(i)}$ is given by the relation [9]

$$a_{(i)}=(\gamma_{ii})^{1/2}a^i=(\gamma_{ii})^{1/2}\sum_{k=1}^{3}\gamma^{ik}a_k. \tag{29}$$

From Eq. (22) we get

$$\gamma_{ik}=\begin{vmatrix}1+\lambda_2^2 & -r\lambda_2 & 0\\ -r\lambda_2 & r^2 & 0\\ 0 & 0 & 1\end{vmatrix}, \tag{30}$$

so that, for instance,

$$d_{(r)}=(1+\lambda_2^2)^{1/2}d^r,\quad d_{(\phi)}=rd^\phi,\quad d_{(z)}=d^z, \tag{31}$$

with similar expressions for the other components. We could of course write the constitutive relations and Maxwell's equations in terms of the usual components $a_{(i)}$ of the electromagnetic field.

IV. DISCUSSION

First note that with the relativistic transformation (2) all the results to be obtained are valid for any value of the parameter $\beta=\Omega R/c$, while with the Galilean transformation (1) one has to assume $\beta\leqslant 1$. However, if $\beta\ll 1$ so that one may neglect the terms in β^2 and higher, then both transformations (1) and (2) give similar results.

Concerning electromagnetism in the corotating and instantaneous inertial frames, the situation is different for the transformations (1) and (2). One has to use in the corotating frame the general covariant formalism of electromagnetism so that constitutive relations as well as Maxwell's equations depend on the metric tensor $g_{\mu\beta}$ in the corotating frame and expression (22) cannot be reduced for any value of the parameter β to the metric tensor obtained with the rotation (1) [see, for instance, the Eq. (7.23) in [9]].

More importantly, for the Galilean transformation (1), the corotating frame is also the instantaneous inertial frame [4,9] so its metric tensor has also to be compared with the metric tensor of the Minkowski space-time valid for the Frenet-Serret tetrad (10). This remark has far-reaching consequences since several problems involving accelerated bodies are more easily solved in the instantaneous inertial frame, for instance, for fields associated with rotating charges (instead of the Lienard-Wiechert potentials). It is clear that using the Frenet-Serret tetrad as the instantaneous inertial frame will give results different from those published in the literature.

V. APPLICATION

As an application of the relativistic description of rotations, we consider the scattering of a harmonic plane wave by a rotating circular cylinder, a problem previously analyzed [17–24] in the frame of Galilean rotations. For an incident E wave

$$E_i(R,\Phi)=E\exp\{-i\omega(T+X/c)\}\mathbf{u}_Z, \tag{32}$$

in which $\mathbf{u}_Z$ is a unit vector in the Z direction and for a dielectric cylinder Maxwell's equations are

$$R^{-1}\partial_\Phi E_Z = -i\omega c^{-1}B_R,$$

$$\partial_R E_Z = i\omega c^{-1}B_\Phi, \tag{33}$$

$$R^{-1}\partial_R(RH_\Phi) - R^{-1}\partial_\Phi H_R = i\omega c^{-1}D_Z.$$

The constitutive relations of the dielectric cylinder in the instantaneous inertial frame are $\mathbf{D}' = \varepsilon\mathbf{E}'$ and $\mathbf{B}' = \mu\mathbf{H}'$, so that the corresponding constitutive relations in the laboratory frame are given by Eqs. (13) (with $\alpha = \lambda = 0$) and (19). Substituting these expressions into Eqs. (33) gives

$$R^{-1}\partial_\Phi E_Z = i\omega c^{-1}[\mu a(s)H_R - b(s)E_Z], \tag{34a}$$

$$\partial_R E_Z = i\omega c^{-1}\mu H_\Phi, \tag{34b}$$

$$R^{-1}\partial_R(RH_\Phi) - R^{-1}\partial_\Phi H_R = i\omega c^{-1}[\varepsilon a(s)E_Z - b(s)H_R], \tag{34c}$$

with

$$a(s) = (1-s^2)(1-n^2s^2)^{-1}, \quad b(s) = m^2 s(1-n^2s^2)^{-1},$$
$$m^2 = n^2 - 1. \tag{35}$$

The derivative with respect to Φ of Eq. (34a) gives

$$R^{-2}\partial_\Phi^2 E_Z - i\omega(cR)^{-1}b(s)\partial_\Phi E_Z = -i\omega\mu(cR)^{-1}a(s)\partial_\Phi H_R, \tag{36a}$$

while from Eqs. (34b) and (34c) we get

$$R^{-1}\partial_\Phi H_R = -i\omega c^{-1}\{c^2/\mu R\partial_R(R\partial_R E_Z) + \varepsilon a(s)E_Z - b(s)H_R\}. \tag{36b}$$

Substituting into Eq. (36b) the expression of H_R taken out from Eq. (34a) gives

$$R^{-1}\partial_\Phi H_R = -i\omega\{[c/\omega\mu R]\partial_R(R\partial_R E_Z) + \varepsilon\omega c^{-1}a(s)E_Z\} - b(s)[\mu a(s)R]^{-1}\partial_\Phi E_Z + i\omega[c\mu a(s)]^{-1}b^2(s)E_Z \tag{36c}$$

and finally substituting Eq. (36c) into Eq. (36a) we get

$$a(s)R^{-1}\partial_R(R\partial_R E_Z) + R^{-2}\partial_\Phi^2 E_Z - 2i[\omega b(s)/cR]\partial_\Phi E_Z + \omega^2c^{-2}[n^2a^2(s) - b^2(s)]E_Z = 0. \tag{37}$$

Now using Eq. (35), a simple calculation gives

$$n^2a^2(s) - b^2(s) = (n^2 - s^2)(1-n^2s^2)^{-1}. \tag{38}$$

Substituting Eqs. (35) and (38) into Eq. (37) and multiplying by $1-n^2s^2$, we get

$$(1-s^2)R^{-1}\partial_R(R\partial_R E_Z) + (1-n^2s^2)R^{-2}\partial_\Phi^2 E_Z - 2i[m^2\omega s/cR]\partial_\Phi E_Z + \omega^2c^{-2}(n^2-s^2)E_Z = 0, \tag{39}$$

which is the equation satisfied by the E_Z component of the electromagnetic field inside the rotating cylinder.

Then, to analyze the scattering of the plane wave (32), we use the Fourier series expansions of $E_Z(R,\Phi)$ and of the incident and diffracted fields $E_{i,Z}(R,\Phi), E_{s,Z}(R,\Phi)$,

$$E_Z(R,\Phi) = \sum A_k E_k(R)\exp(ik\Phi), \tag{40a}$$

$$E_{i,Z}(R,\Phi) = E\sum i^{-k}J_k(R)\exp(ik\Phi),$$

$$E_{s,Z}(R,\Phi) = \sum B_k H_k^{(2)}(R)\exp(ik\Phi), \tag{40b}$$

in which J_k and $H_k^{(2)}$ are the usual Bessel and Hankel functions. The boundary conditions requiring that E_Z and $\partial_R E_Z$ are continuous at the surface $R=a$ of the cylinder give two sets of equations supplying the unknown coefficients A_k and B_k. Substituting Eq. (40a) into Eq. (39) gives, for $E_k(R)$, the differential equation

$$(1-s^2)R^{-1}\partial_R(R\partial_R E_k) + [\omega^2c^{-2}(n^2-s^2) + 2\omega m^2ks/cR - (1-n^2s^2)k^2R^{-2}]E_k = 0 \tag{41}$$

to be compared [see Eq. (10.71) in [9]] with the equation supplied by Galilean rotations

$$R^{-1}\partial_R(R\partial_R E_k) + [\omega^2c^{-2}\varepsilon + 2\omega\Omega c^{-2}k(\varepsilon-1) - k^2R^{-2}]E_k = 0. \tag{42}$$

The rotation of the cylinder breaks the symmetry of the scattering pattern and generates a distortion that can be used to probe the structure of the cylinder (it could be, for instance, a plasma column), so it is important to get a reliable approximation for $E_k(R)$. Since the relativistic covariance of Maxwell's equations is not satisfied with Galilean rotations, it is not sure that the Bessel equation (42) supplies such an approximation. So we are left with Eq. (41), which is difficult to solve since $s = \tanh(\Omega R/c)$ is a function of R and particular methods of approximations have to be used. The previous analysis can be extended to H waves and to waves impinging at oblique incidence. We get slightly more intricate equations for conducting cylinders, especially in the case of H waves for which the boundary conditions require some attention. In the future we plan to discuss these problems as well as the electromagnetic analog of the Magnus effect [25,26] with a comparison of the results obtained in previous works for cylinders rotating with a small angular velocity [18,24].

There is an interesting situation with which Galilean rotations do not cope. Consider a good conducting cylinder such as the electromagnetic field, which is concentrated inside a thin skin sheet bounded by the classical skin depth.

Suppose that this cylinder is spinning with a large angular velocity so that the azimuthal velocity in the skin sheet is nearly equal to the velocity of light c. What happens when this velocity tends to c? We plan to prove that the cylinder behaves like a mirror [27].

It should be mentioned that it has been claimed [28] that linear, nonreciprocal biisotropic media [that is, media with constitutive relations (5)] are forbidden and that one should have $\alpha+\lambda=0$. However, this statement is still controversial [29–31]. Of course, when $\alpha+\lambda=0$ some of the previous results simplify. For recent works on electrodynamics of moving chiral media, see [32,33].

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